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Steady state temperature of a two-layer body subjected to a moving heat source and convective cooling

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ABSTRACT

Surface coatings are often used in tribological applications to protect the surfaces subject to friction. Coatings can play a double role: (i) improvement of the resistance to wear, (ii) enhancement of the diffusion of heat generated by friction. We present in this work an analytical modelling of the three-dimensional heat diffusion in a two-layer medium subjected to local heating and to surface cooling. The solid is moving with respect to the heat source that causes the heating. Based on this model, we study the variation of the temperature distribution as a function of physical parameters such as: (i) the coating thickness, (ii) the relative velocity of the solid with respect to the heat source (iii) the convective heat transfer coefficient.

1. Introduction

Surface coatings are regularly used in tribology to ensure mechanical protection of solids subjected to friction, as it is the case of bearings, drive shafts, gears, etc. These coatings are often of very small thickness varying from a few to one hundred microns. They play a mechanical role, but also an important thermal function. Effectively, when the coating is a good thermal conductor (like, for example: silver, molybdenum or nickel) it ensures a more effective diffusion of heat than a non-coated substrate made of steel, the material most commonly used in mechanics.

The study of heating due to friction is the object of many research works. Some authors present the case of a semi-infinite thermally isolated solid subjected to a heat source [1–5] or to several sources [6]. Some other works introduce the surface cooling and analyse its influence on the temperature distribution [7–12].

In practice, friction implies at least two solids. Heat transfer is then coupled between the solids by a thermal contact resistance [13–20].

The present work deals with the problem of a two-layer medium, subject to a local, moving, rectangular heat source and cooled by convection. The medium considered has finite dimensions, what makes it possible to study the influence of the geometry on its thermal behaviour. The analytical solution proposed serves to calculate the three-dimensional distribution of temperature in the medium without any restriction in terms of dimensions, relative speed or surface cooling. We present the details of the analytical solution developed and analyse the thermal behaviour of the solid according to the coating thickness, the velocity and the convection heat transfer coefficient.

2. The studied problem

The studied medium consists of a substrate (2) and a coating (1) as shown in Fig. 1. Dimensions are finite in the x-y plane ($2A$ in x-direction and $2B$ in y-direction). The coating and substrate thicknesses are denoted by d_1 and d_2 respectively. The medium is subject to a uniform rectangular heat source of dimensions $2a$ in x-direction and $2b$ in y-direction. Medium and source are moving with respect to each other with a constant relative velocity V in x-direction. Coating and substrate are considered to be in perfect contact. The back face of the substrate is maintained at a reference temperature $T = 0$. The upper face of the coating ($z = -d_1$) is cooled by convection (with a convection coefficient h and free-stream temperature $T = 0$) while faces $y = -B$ and $y = B$ are considered adiabatic. Thermal properties of the coating and substrate are supposed to be independent of temperature. The surface heat flux being uniform, it is possible to use the condition of symmetry in $y = 0$. Faces $x = -A$ and $x = A$ are subject to periodic boundary conditions. This last condition

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Nomenclature

A	Semi-length of the domain in x-direction, m
a	Semi-length of the heated area in x-direction, m
B	Semi-length of the domain in y-direction, m
b	Semi-length of the heated area in y-direction, m
d	Thickness, m
h	Heat convection coefficient, W.m^{-2}.K^{-1}
q	Heat flux density, W.m^{-2}
T	Temperature rise, °C
V	Velocity, m.s^{-1}
x, y, z	Cartesian coordinates, m
Greek Symbols	
α	Thermal diffusivity, m^2.s^{-1}
λ	Thermal conductivity, W.m^{-1}.K^{-1}
Subscript	
$1, 2$	Coating: 1, substrate: 2
Superscript	
$*$	Dimensionless quantity
$\sim, \approx$	Integral transforms

corresponds to the practical case of cylindrical rings in rotation with respect to the source (bearings, drive shafts, rolling mills, etc.).

3. The model

Taking into account the boundary conditions described above and the notation used in Fig. 1, the heat diffusion in the coating and substrate is governed by the following equations

(i) 3D diffusion in the coating (1) and substrate (2)

$$\frac{\partial^2 T_j}{\partial x^2} + \frac{\partial^2 T_j}{\partial y^2} + \frac{\partial^2 T_j}{\partial z^2} - \frac{V}{\alpha_j}\frac{\partial T_j}{\partial x} = 0, \quad j = 1,2 \tag{1}$$

(ii) periodic conditions in x-direction

$$(T_j)_{x=-A} = (T_j)_{x=A};\ \left(\frac{\partial T_j}{\partial x}\right)_{x=-A} = \left(\frac{\partial T_j}{\partial x}\right)_{x=A} \tag{2}$$

(iii) boundary condition in y-direction

$$\left(\frac{\partial T_j}{\partial y}\right)_{y=0} = 0;\ \left(\frac{\partial T_j}{\partial y}\right)_{y=B} = 0 \tag{3}$$

(iv) boundary condition in z-direction

$$-\lambda_1\left(\frac{\partial T_1}{\partial z}\right)_{z=-d_1} = \begin{cases} q & (|x|\le a, |y|\le b) \\ -h(T_1)_{z=-d_1} & (|x|\le A, |y|\le B) \end{cases};\ (T_2)_{z=d_2} = 0 \tag{4}$$

(v) interface coating-substrate

$$-\lambda_1\left(\frac{\partial T_1}{\partial z}\right)_{z=0} = -\lambda_2\left(\frac{\partial T_2}{\partial z}\right)_{z=0};\ (T_1)_{z=0} = (T_2)_{z=0} \tag{5}$$

Equation (1) is written in a reference frame attached to the heat source, what explains the presence of the transport (or convection) term $V/\alpha_j \partial T_j/\partial x$.

With the periodic and symmetry boundary conditions given by Equation (2) and Equation (3), the integral frequency and the finite cosine Fourier transforms can be used respectively expressed as (see [4] and [21] for details)

$$\tilde{T}_j = \frac{1}{2A}\int_{-A}^{A} T_j e^{-im\pi x/A} dx;\ \tilde{\tilde{T}}_j = \frac{1}{B}\int_0^B \tilde{T}_j \cos\left(\frac{n\pi y}{B}\right) dy \tag{6}$$

where i is the imaginary unit number.

Transformed equations are put into the form

$$\frac{d^2\tilde{\tilde{T}}_{j,mn}}{dz^2} - \gamma_{j,mn}^2 \tilde{\tilde{T}}_{j,mn} = 0 \tag{1a}$$

$$-\lambda_1\left(\frac{d\tilde{\tilde{T}}_{1,mn}}{dz}\right)_{z=-d_1} = \tilde{\tilde{q}}_{mn} - h\left(\tilde{\tilde{T}}_{1,mn}\right)_{z=-d_1};\ \left(\tilde{\tilde{T}}_{2,mn}\right)_{z=d_2} = 0 \tag{4a}$$

$$-\lambda_1\left(\frac{d\tilde{\tilde{T}}_{1,mn}}{dz}\right)_{z=0} = -\lambda_2\left(\frac{d\tilde{\tilde{T}}_{2,mn}}{dz}\right)_{z=0};\ \left(\tilde{\tilde{T}}_{1,mn}\right)_{z=0} = \left(\tilde{\tilde{T}}_{2,mn}\right)_{z=0} \tag{5a}$$

The expressions of $\gamma_{j,mn}$ and $\tilde{\tilde{q}}_{mn}$ are given by Equation (7).

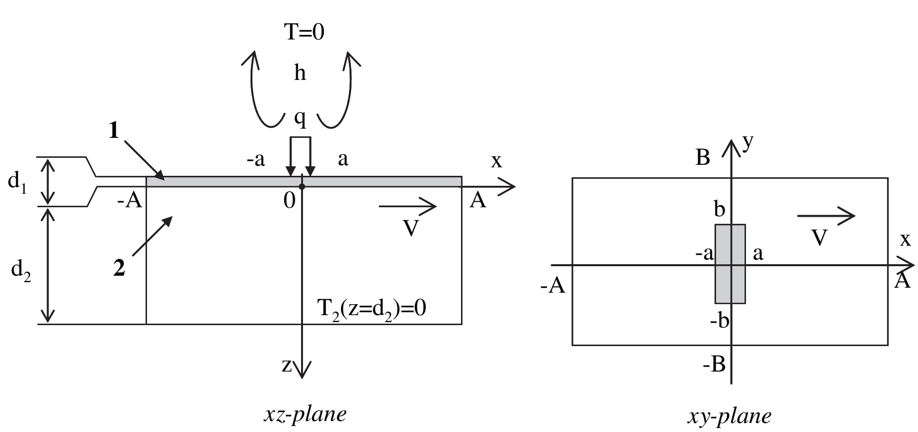

Fig. 1. Two-layer medium (parameterisation).

Table 1
Thermal properties of materials.

Material	Thermal conductivity (W/m.K)	Thermal diffusivity (m^2/s)
Silver (1)	430	18×10^{-5}
Steel (2)	50	0.5×10^{-5}

Table 2
Dimensions adopted for the studied medium.

A (m)	B (m)	a (m)	b (m)	d_2 (m)
0.01	0.01	0.0005	0.0005	0.03

To solve Equation (1a) it is advisable to distinguish the case $m = 0$ and $n = 0$ (i.e. $\gamma_{00} = 0$) from the rest of cases: $m \neq 0$ and/or $n \neq 0$ (i.e. $\gamma_{mn} \neq 0$). The solutions of these two cases are given respectively by Equations (1b) and (1c) as follows

$$\tilde{\tilde{T}}_{j,00} = A_{j,00} z + B_{j,00} \quad (\text{for } m = 0 \textit{ and } n = 0) \tag{1b}$$

$$\tilde{\tilde{T}}_{j,mn} = A_{j,mn}\mathrm{ch}(\gamma_{j,mn} z) + B_{j,mn}\mathrm{sh}(\gamma_{j,mn} z) \quad (\text{for } m \neq 0 \text{ and/or } n \neq 0) \tag{1c}$$

The constants $A_{j,00}$, $B_{j,00}$, $A_{j,mn}$ and $B_{j,mn}$ in previous equations are determined from the boundary conditions (4a) and (5a), and are expressed as

$$\begin{aligned}
&A_{1,00} = \frac{-\tilde{\tilde{q}}_{00}}{D_{00}};\ B_{1,00} = -\frac{\lambda_1}{\lambda_2} d_2 A_{1,00};\ A_{2,00} = \frac{\lambda_1}{\lambda_2} A_{1,00};\\
&B_{2,00} = -\frac{\lambda_1}{\lambda_2} d_2 A_{1,00};\ D_{00} = \lambda_1 + h\, d_1 + h\, d_2\, \lambda_1/\lambda_2;\ \tilde{\tilde{q}}_{00} = q\frac{a\, b}{A\, B};\\
&A_{1,mn} = \frac{\tilde{\tilde{q}}_{mn}}{D_{mn}};\ B_{1,mn} = -K_{mn} A_{1,mn};\ A_{2,mn} = A_{1,mn};\\
&B_{2,mn} = -\frac{\lambda_1 \gamma_{1,mn}}{\lambda_2 \gamma_{2,mn}} K_{mn} A_{1,mn};\\
&D_{mn} = \lambda_1 \gamma_{1,mn}\left[\mathrm{sh}(\gamma_{1,mn} d_1) + K_{mn}\mathrm{ch}(\gamma_{1,mn} d_1)\right]\\
&\qquad + h\left[\mathrm{ch}(\gamma_{1,mn} d_1) + K_{mn}\mathrm{sh}(\gamma_{1,mn} d_1)\right];\\
&K_{mn} = \frac{\lambda_2 \gamma_{2,mn}\mathrm{ch}(\gamma_{2,mn} d_2)}{\lambda_1 \gamma_{1,mn}\mathrm{sh}(\gamma_{2,mn} d_2)};\\
&\gamma_{j,mn} = \left[\left(\frac{m\pi}{A}\right)^2 + \left(\frac{n\pi}{B}\right)^2 + i\left(\frac{m\pi}{A}\right)\frac{V}{\alpha_j}\right]^{1/2};\\
&\tilde{\tilde{q}}_{mn} = q\frac{\sin(m\pi a/A)\sin(n\pi b/B)}{(m\pi)\,(n\pi)}
\end{aligned} \tag{7}$$

The constants K_{mn}, $A_{j,mn}$, $B_{j,mn}$ given by Equation (7) depend on the ratio $\lambda_2\gamma_{2,mn}/\lambda_1\gamma_{1,mn}$. Taking into account the dependence of $\gamma_{j,mn}$ on α_j, it becomes apparent the incidence of the thermal effusivities of both solids on the obtained solution.

The inverse transforms of Equations (6) are

$$\begin{aligned}
T_j(x,y,z) &= \sum_{m=0}^{\infty} c_m \Re\left\{\tilde{T}_{j,m}(y,z) e^{im\pi x/A}\right\};\\
\tilde{T}_{j,m}(y,z) &= \sum_{n=0}^{\infty} c_n \tilde{\tilde{T}}_{j,mn}(z)\cos\left(\frac{n\pi y}{B}\right)
\end{aligned} \tag{6a}$$

with $c_0 = 1$ and $c_{m,n \neq 0} = 2$.

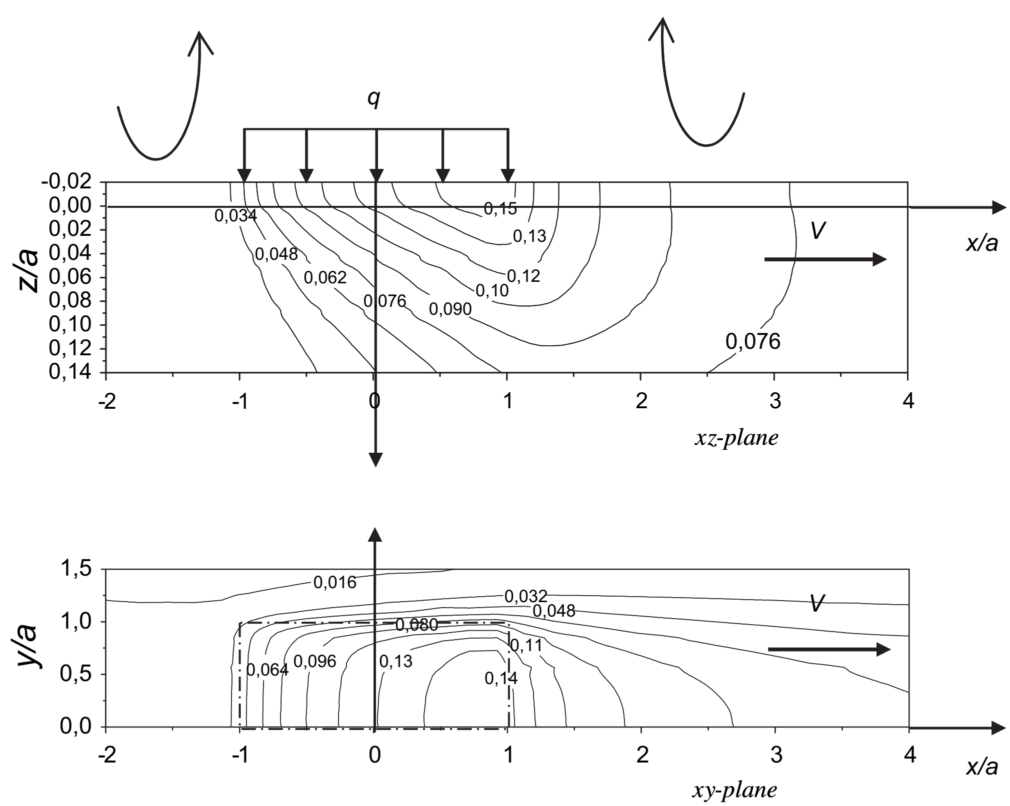

Fig. 2. Isotherms, $T^* = T/(qA/\lambda_2)$, in the x-z and x-y planes for $h^* = 6$, $V^* = 100$ and $d_1^* = 0.02$.

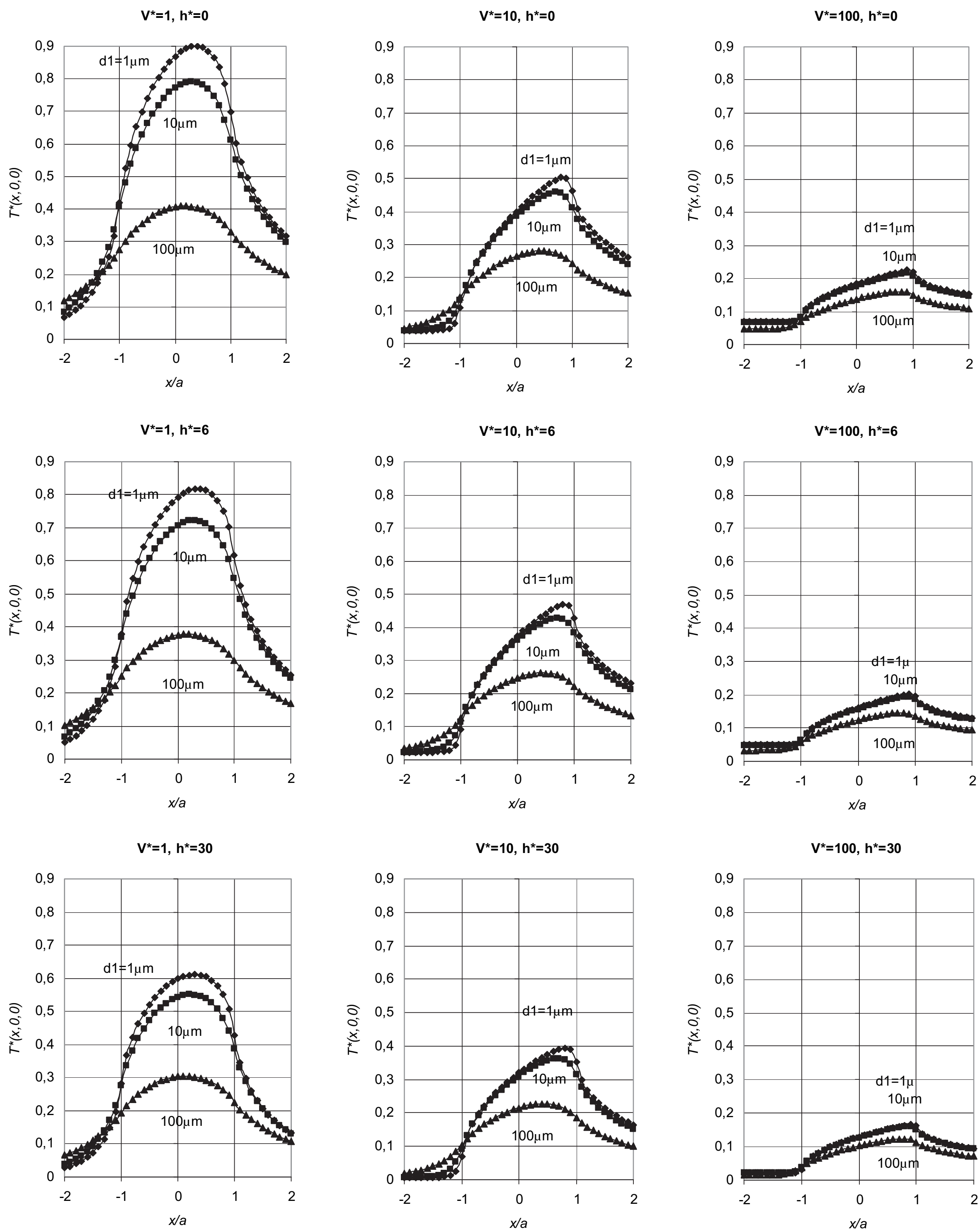

Fig. 3. Evolution of the dimensionless surface temperature $T^*(x, 0, -d_1)$ for different values of velocity V, coating thickness d_1 and convection coefficient h.

4. Results and discussion

Osman et al. [10] recently developed an analytical solution for a homogeneous medium, under similar conditions to ours, but without coating. By adopting equal thermal properties for the film and substrate, we have verified that our solution becomes identical to that found by those authors.

Furthermore, we develop our solution for a particular case where the medium is steel covered by a layer of silver. The thermal properties of both materials are gathered in Table 1.

We have fixed the dimensions A, B, a, b and d_2 to focus the analysis on the effect of the coating thickness d_1, the speed V and the convection heat transfer coefficient h. Dimensions used are given in Table 2. The heat flux density is $q = 10^5$ W.m^{-2}.

The number of terms used to ensure the convergence of the series in Equation (6a) depends on the ratio ab/AB. The smaller is this ratio the larger the number of terms to be taken. For the case treated in this study, the number of terms is 500.

Fig. 2 shows an example of thermal maps (isotherms) in the planes x-z ($y = 0$) and x-y ($z = -d_1$) corresponding to a zoom of the hottest part of the studied case. The dimensionless spatial coordinates used are referred to the source width a. Temperatures are also expressed in terms of the dimensionless variable $T^* = T/(qA/\lambda_2)$. The marked line corresponds to the coating-substrate interface. From the thermal cartography in the x-z plane it is observed a deformation of the isotherms in the direction of the movement. The isotherms at higher temperatures are located on the side of the source's exit in the direction of the relative motion with respect to the solid. The thermal cartography in the $x-y$ plane ($z = -d_1$) shows also a deformation of the isotherms following the geometry of the source (the source is localised at $-1 \le x/a \le 1$ and $0 \le y/a \le 1$) on the upstream side and gradually forms a thermal trail on the downstream side. There is a factor 10 between the highest dimensionless temperature, 0.14, and the smallest one, 0.016, although they are located relatively close (approximately 1.5 times the size of the source a).

To study the temperature changes with the physical parameters of the problem, we have considered the following values:

- Dimensionless thicknesses of silver $d_1^* = d_1/a$: 0.002, 0.02 and 0.2 (corresponding to film thicknesses $d_1 = 1$ μm, 10 μm and 100 μm respectively)
- Dimensionless velocity $V^* = Va/\alpha_2$: 1, 10 and 100
- Dimensionless convection heat transfer coefficient $h^* = hd_2/\lambda_2$: 0, 6 and 30

The dimensionless quantities h^* and V^* are equivalent to the Biot number and Peclet number respectively.

These values cover a broad range of parameters met in practical cases. We have put together on the same figure (Fig. 3) the results corresponding to the 9 combinations of these parameters. The graphs show the evolution of the dimensionless surface temperature $T^*(x, 0, -d_1)$with the abscise x in the neighbourhood of the heat source.

For fixed convection coefficient and coating thickness, the increase of velocity tends to flatten the amplitude of temperature variations and to move the maximum towards the exit of the contact. For given values of velocity and convection coefficient, the increase in silver thickness induces a reduction in temperature values and a displacement of its maximum towards the opposite direction to the movement. It is the reverse behaviour compared to that of the velocity increase. In effect, the thermal response is conditioned by the value of the Peclet number $V^* = Va/\alpha$, so when the thickness of the silver layer increases, its thermal diffusivity (greater than that of steel) has a larger influence and the value of V^*decreases.

It is well-known that the smaller the Peclet number, the closer is the maximum of temperature to the source centre. The limit case $V^* = 0$ corresponds to the static solid for which the maximum of temperature is at the centre of the contact.

The coating plays an important role in the temperature level reached particularly at very low velocities. When the velocity is important, a relatively small coating thickness can be adopted. In fact, when the velocity increases, the thermal penetration depth decreases. It is thus not necessary to put a thick coating layer for high speeds. This feature is independent of the value of the convection coefficient that characterises the heat dissipation. Variations of the heat transfer coefficient affect all the temperatures in the same proportion. Contrary to the effect produced by the variations of film thickness and velocity, those of the convection coefficient do not modify the appearance of the temperature profile, but only the temperature level.

5. Conclusions

The analytical solution developed in the present work makes it possible to easily calculate the three-dimensional temperature distribution in a two-layer solid subject to a moving heat source. It permits to overcome the difficulties inherent to numerical modelling for which special precautions must be taken, in particular in the choice of a suitable grid according to the size of the heat source and the relative velocity. The results obtained in the case of a steel substrate covered with a layer of silver yields the following physical tendencies: (i) The increase in velocity involves a reduction in the temperature amplitude and a displacement of the temperature maximum in the direction of the movement (ii) The increase in the thickness of silver film involves a reduction in the temperature and a displacement of the temperature maximum towards the opposite direction to the movement (iii) The increase in the convection heat transfer coefficient acts only on the overall temperature level, but does not modify the temperature profiles.

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